Chapter 5. Estimation of Design Resistance and development of Interaction Curves

5.1 Introduction

This chapter deals with the calculation of the ultimate moment of resistance of the Reinforced Concrete tubular section of the tower. There are many methods prescribed in the codes for the purpose of estimation of the ultimate loads. These methods differ primarily with regard to the model used to represent the stress strain curve of concrete in compression.

The ultimate moment capacity of the tubular Reinforced Concrete section depends on the normal compressive load that acts at that point. The interaction of this normal force with the ultimate moment, corresponds particularly to the location of the neutral axis which generally falls within the section for the high eccentricities in loading usually encountered under extreme wind speeds.

The following are some of the assumptions commonly adopted for the purpose of estimation.

1. Place sections remain plane after bending. This means that a linear strain distribution is assumed at the cross section.
2. Extreme fibre stresses are computed at the center line of the concrete shell. The mean radius is representative of all stresses.
3. The vertical reinforcing steel is replaced by an equivalent thin steel shell, located at the mean radius.
4. The stress-strain relationship of steel is assumed to be elasto-plastic, and is assumed to be identical in tension and compression.
5. Tensile stresses in concrete are ignored. The section is assumed to be fully cracked in the tension region of the neutral axis.

In addition, the following are some requirements before the calculations can be done.

- Stress-strain relationship of concrete in compression
- Limiting compressive strain in concrete

- Limiting tensile strain in steel
- Modulus of elasticity of steel

The differences in the various codal methods are basically caused due to dissimilarities in the above assumptions.

This paper calculates the design resistance using the standard stress-strain curve for steel and a proposed stress-strain curve for concrete. This curve was proposed by Dr. Devdas Menon in his Ph.D. thesis.

5.2 Characteristic Stress-Strain Curve for Steel

The stress-strain curve for steel is more or less standard and is used by all the codal provisions. It is an idealized elasto-plastic relationship. The values to be assumed are the E_s (modulus of elasticity for steel) and the ε_{sml} (limiting tensile strain in steel).

A diagrammatical representation of the Steel stress-strain curve is given below

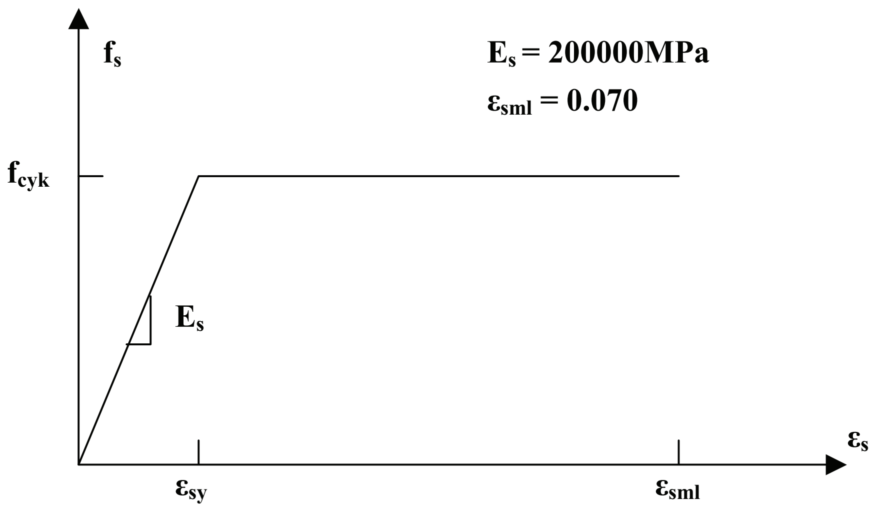

Figure 5.1 – Stress-strain curve (steel)

As has been indicted the value of

E_s = 200,000 N/mm2

ε_{sml} = 0.07 (as initially proposed by the ACI code)

The value for the limiting tensile strain is assumed for some codes to be a very conservative 0.05. This is probably to take care of the excessive cracking in concrete on the tension side. This however is not strictly called for at ultimate loads, in the limit state of collapse, since the crack control is checked for separately as part of the serviceability requirements.

5.3 Characteristic stress-strain curve for concrete

Various codes give various stress-strain codes for concrete.

The ACI code for example employs the Hognestad's curve, originally proposed for eccentrically loaded columns. The curve has two parts. The first is a parabolic curve and the second is a straight line that continues from the end of the parabolic curve that represents the downward trend of the curve. It assumes a limiting strain under direct compression of 0.002 and an ultimate strain in flexure of 0.003.

On the other hand, the CICIND has a very elaborate curve. It is a parabolic-linear curve that distinguishes between the effects of dynamic, short-term loading and static long-term loading.

The curve that is used for the purpose of estimation of resistance and for the purpose of generation of the interaction curves is a new curve. This curve has been proposed taking into account the effect of tubular geometry and the effect of short-term wind loading.

The limiting compressive strain in concrete ε_{cul} corresponds to the maximum value of the strain ε_{cu} at the middle of the concrete shell thickness at the extremity of compression. Since the shell is extremely thin in comparison to its very large diameter, the distribution of stress across the thickness of the shell is almost uniform. The behavior of thin walled chimneys is very different from the behavior of solid Reinforced Concrete sections which can accommodate a large strain variation across the cross section.

Hence the value of ε_{cul} should not be as large as 0.003 as suggested by the codes. Rather it must be restricted to a value usually specified under conditions of uniform compression, that is $\varepsilon_{cul} = 0.002$.

The CICIND code proposition of distinctively accounting for the dynamic short-term loading effect of wind merits consideration. However the premises on which the curve is based are questionable. It is, for example, observed that the wind loads are extremely short-lasting, while the meteorological practice is to compile hourly mean wind speeds. The values for the code are taken from practical tests where the loading was done by reversed cyclic bending. However since the dynamic nature of wind consists of random velocity fluctuations about a mean, rather than complete change of direction in short periods. Since the mean response to wind loading is fairly substantial and the overall response is quasi-static in nature, the behavior is better approximated by monotonic loading rather that reversed cyclic loading; the duration of the loading to be considered is approximately 2 to 5 hours.

On the basis of the results of a large number of tests on eccentrically loaded concrete cylinders under varying load conditions the following conclusions can be drawn

- The stress strain curve is parabolic rather than linear, even under the short term loading under consideration.
- If f_{cu} = 0.85 f'_{ck} is assumed then it is reasonable to assume an increase of approximately 10% for relatively short time loading.
- The value of the ultimate compressive strength ε_{cul} corresponding to this peak may be assumed to be approximately 0.002 for both short-term and long-term loading.

On the basis of the above discussion the following curve is assumed as the stress-strain curve for concrete under compression. It employs a simple parabolic curve with a limiting ultimate limiting strain of 0.002 and a value of f_{cu} = (0.85 f'_{ck}) C_S. Here the term C_S is called the short term loading factor, having a value that depends on the normal compression on the tower section; it is assumed to vary linearly between a maximum value of (0.95/0.85) for normal load = 0 and to unity when the value of normal load is maximum – that is under pure compression.

The formula for the curve is given below

$$f_{pc1} = C_s f_{pc} \quad \textbf{(5.1)}$$

where

$$C_s = \frac{\left\{0.95 - 0.1\dfrac{N}{N_{max}}\right\}}{0.85} \quad \textbf{(5.2)}$$

The curve is as shown in the following figure.

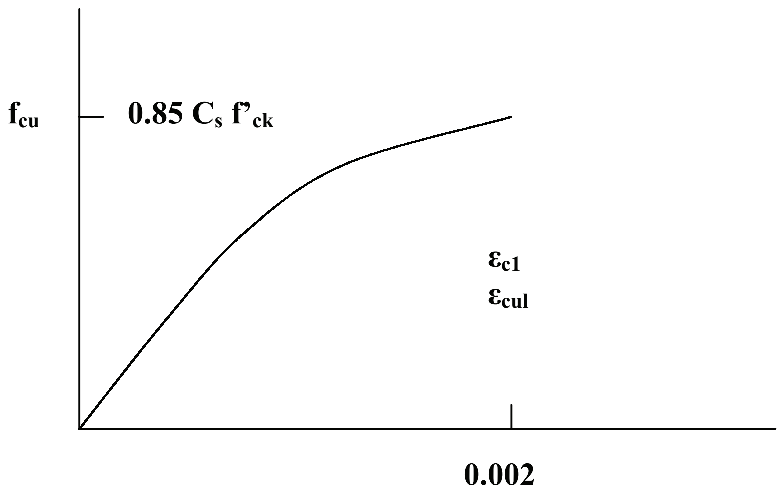

Figure 5.2 – Stress-strain curve (Concrete)

Design Stress-Strain Curve

The characteristic stress-strain curve refers to the 'actual' characteristic values of the stress-strain values. These are multiplied by the partial safety factors to get the design curves. The values of the partial safety factors assumed are as follows

$\gamma_s = 1.15$

$\gamma_c = 1.50$

these design curves are used to calculate the design ultimate moment carrying capacity of the Reinforced Concrete tubular section.

The codes also specify either the design or the characteristic curves. The CICIND code for example specifies the design curves along with the characteristic curves whereas the ACI method specifies the design curve which is to be multiplied with a 'resistance factor' of 0.8. The code does not recommend any 'design stress-strain curves'.

5.4 Calculation of Ultimate moments

The ultimate moment carrying capacity M_u of tubular section, corresponding to any given normal compression N is determined by solving the following equilibrium equations.

$$N = N_c + N_s \quad \textbf{(5.3)}$$

$$M_u = M_{uc} + M_{us} \quad \textbf{(5.4)}$$

where N_c and N_s are the resultant normal forces obtained from the concrete and steel stress blocks respectively. M_{uc} and M_{us} denote the respective moments of the concrete and steel blocks about the centerline.

The following diagram is a representation of the various components involved in the estimation of the design interaction curves.

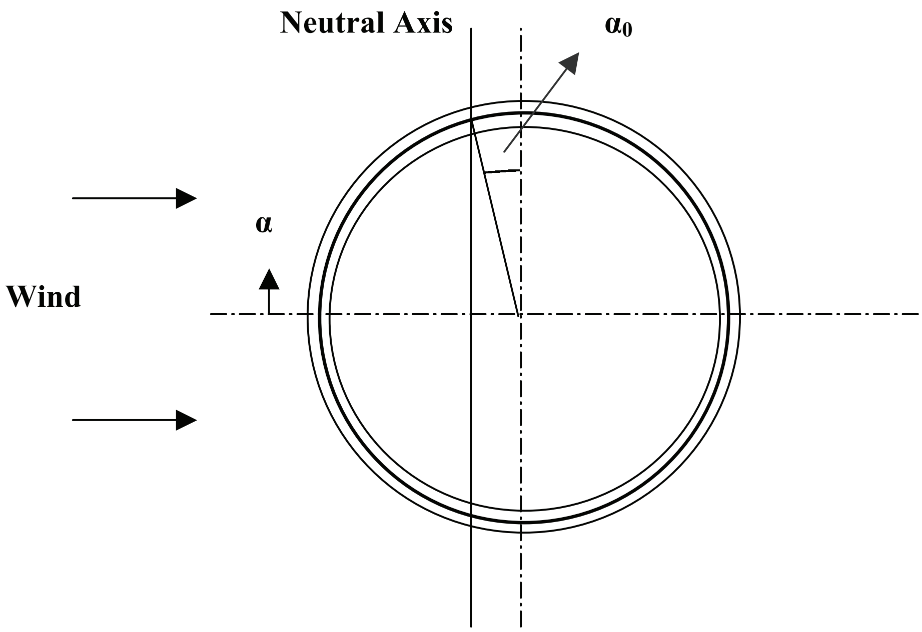

Figure 5.3 – Chimney Cross section

The distribution of strains and the corresponding stresses are given in the below

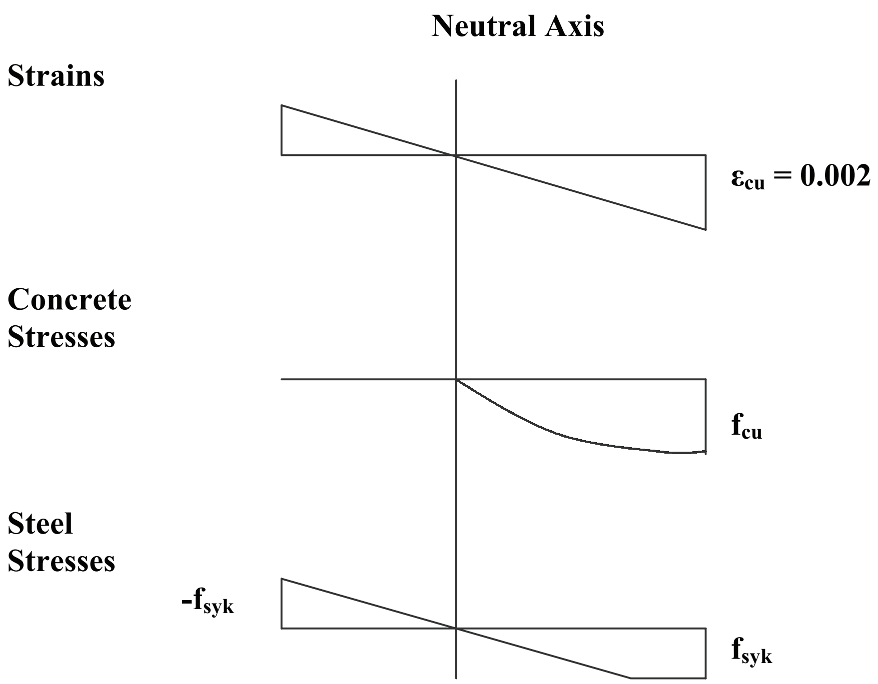

Figure 5.4 – Stress and strain distributions

These diagrams are merely depictive. They do not show the actual values.

As can be seen from the diagrams, for a neutral axis there exists a strain distribution. This strain distribution is linear because of the assumption we had made in the starting of the chapter. This in turn determines the stresses in the concrete and steel block. The summation of these stresses gives rise to the resistive strength of the chimneys.

5.5 Interaction Curve

The interaction curve is a complete graphical representation of the design strength of a Reinforced Concrete chimney. Each point on the curve corresponds to the design strength values of N and M_u. That is to say that if the load of N were to be applied to the Reinforced Concrete chimney with an increasing eccentricity then the value of the eccentricity where this line would intersect with the interaction curve is given by

$$\varepsilon = \frac{Mu}{N} \qquad \textbf{(5.5)}$$

The interaction curve is the failure envelope. Any point inside the curve is 'safe'. That is any combination of moment and compressive strength where the point lies within the curve will not cause failure of the Reinforced Concrete chimney.

In reality the loading is not done in this manner. Given values the moment and the compressive stress, it should be possible to check whether the chimney cross section is safe.

The magnitude of N determines the neutral axis. This location is specified by the angle α_0 in the equation and the diagram given above. On location of the neutral axis the strain distribution is known. This can then be used to solve for the value of N and the ultimate moment M_u. It is therefore obvious that the solution to the above set of equations can be found as a closed form solution. This is because the location of the neutral axis is required for the calculation of the normal force N, while the value of N is itself required for the location of the neural axis.

For the purpose of developing the interaction curves the location the neutral axis was assumed and the values of the normal force and the moment were calculated. The neutral axis was then changed to calculate a new set of N and M_u. This was repeated to get the interaction curves of N Vs M_u.

Not all locations of the neutral axes are realistically feasible, as will be seen in the following discussion.

The following diagram depicts the variation of the strain profile with change in the location of the neutral axis.

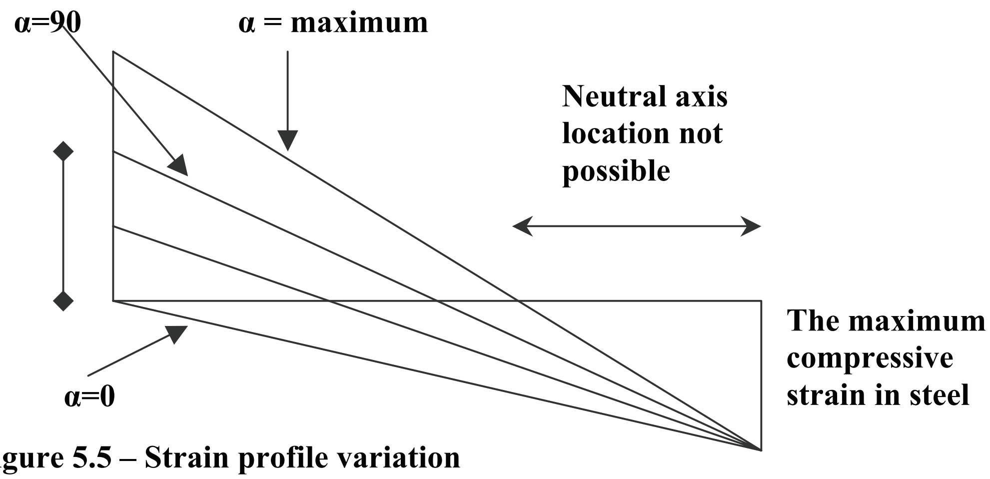

Figure 5.5 – Strain profile variation

As the angle that locates the neutral axis α changes from 0 the location of the neutral and hence the participation of steel in taking the load varies. This continues as more and more participation of steel in tension occurs and the net compressive force on the chimney reduces. At a particular value of α the value of steel in tension effectively nullifies the effect of the compression of the concrete block. Any increase in the value of α is not possible because it follows that the chimney in overall tension, which is not possible.

Although the interaction curve is plotted between the value of N and M_u, in the interest of greater flexibility, the interaction curve is rendered non dimensional by use of the following relations

$$n = \frac{N}{f'_{ck}\, rt} \qquad \textbf{(5.6)}$$

$$m = \frac{M_u}{f'_{ck}\, r^2 t} \qquad \textbf{(5.7)}$$

Where r is the value of the radius of the section in consideration of the Reinforced Concrete chimney, and t is the thickness of the section.

5.5.1 Family of interaction curves

Since we are using the non dimensional parameters m and n, the curves are no longer applicable to one chimney alone. It is possible to plot a family of curves that vary with respect to one parameter. Once the parameter value is known, it is possible to calculate the corresponding value for any new chimney and then reuse these curves for that particular chimney.

The parameter that was used for the purpose of generating a family of curves was

$$\rho \frac{f_{syk}}{f'_{ck}} \qquad \textbf{(5.8)}$$

Where

ρ is the percentage of steel

f_{syk} and f'_{ck} are the strengths of steel and concrete.

A program was written in C++ that was used to that calculate the values of pairs of values of n and m. The iteration was done by varying the value of the angle of the neutral axis in incremental steps of 1 degree. Then the strain distribution for that particular neutral axis was evaluated. The total force contributed by the concrete and steel sections was evaluated by integration. Then the value obtained was non-dimensionalised using the factors as appropriate. This was continued till the value of the total normal force evaluated to zero, signaling that the limit of the neutral axis was achieved. The program listing is given in the appendix.

The interaction curve is given below.

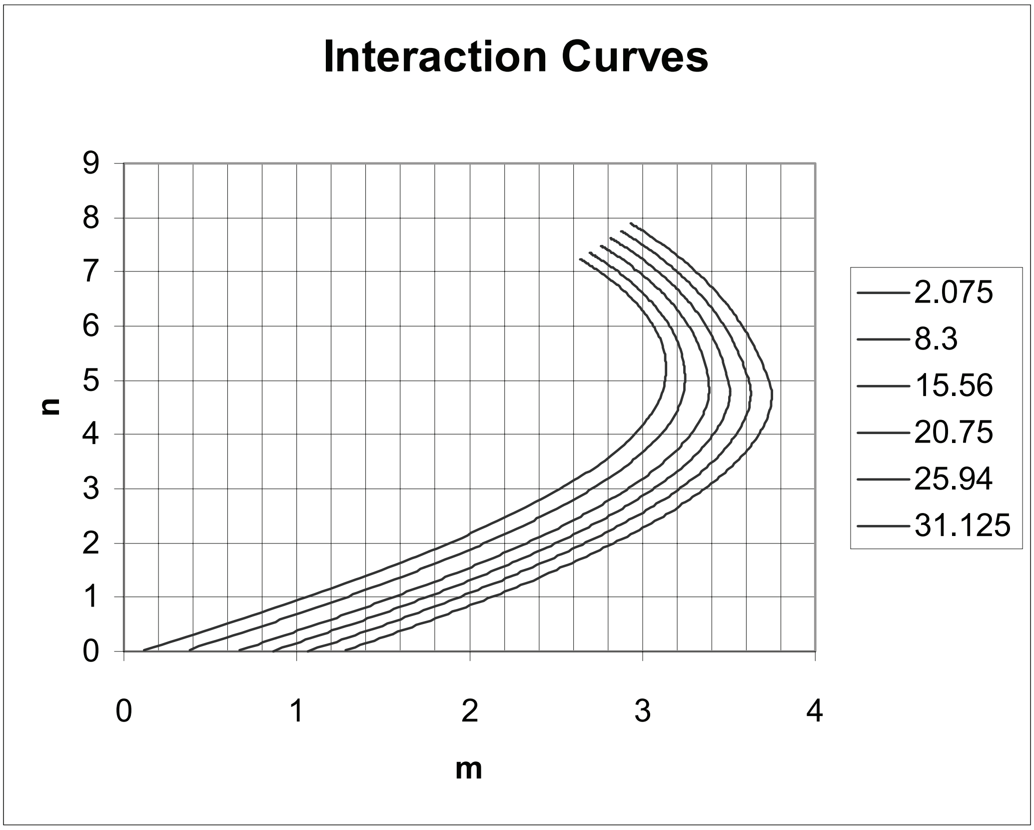

Figure 5.6 – Interaction curves

The family of curves is from the parametric variation of the term given earlier. The values of the parameter for each of the curve is given in the chart. The curves have to be read left to right. That is the first curve on the left refers to the value

$$\rho \frac{f_{syk}}{f'_{ck}} = 2.075 \quad \textbf{(5.9)}$$

And so on.

The values of the terms utilized to arrive at the values are given below

ρ	f'_{ck}	f_{syk}	$\rho(f_{syk}/f'_{ck})$
0.2	40	415	2.075
0.8	40	415	8.3
1.5	40	415	15.5625
1.5	30	415	20.75
1.5	24	415	25.94
1.5	20	415	31.125

Table 5.1 – Values of the interaction curve parameter

From the table the ranges assumed for the values are also visible. The percentage of steel is assumed from 0.2% to 1.5% which is the normal range. The value of f'_{ck} too is assumed to be varying from 20 to 40, that is use of concrete of grades M20 to M40 has been assumed.

The usage of these curves for the estimation of strength is shown in the chapter "Design and detailing of Example Chimney".

5.5.2 Derivation of equations used

The derivation of the equations for the calculation is given below.

$$fcu = 0.85 C_s f'_{ck} \quad \textbf{(5.10)}$$

Where

C_s is the short term loading factor that varies linearly as explained earlier.

$\varepsilon_{cul} = 0.002$

The stress-strain curve for concrete is given below

$$f_{pc} = \frac{f_{cu}}{\gamma_c}\left\{2\left(\frac{\varepsilon}{\varepsilon_{c1}}\right)-\left(\frac{\varepsilon}{\varepsilon_{c1}}\right)^2\right\} \quad \textbf{(5.11)}$$

The stress-strain curve for steel is given below

$$fs = \begin{cases} E_s\varepsilon \rightarrow -\varepsilon_{sy} \le \varepsilon \le \varepsilon_{sy} \\ fsyk\left(\frac{\varepsilon}{|\varepsilon|}\right) \rightarrow \varepsilon_{sy} < |\varepsilon| \end{cases} \quad \textbf{(5.12)}$$

Where

$$\varepsilon_{sy} = \frac{f_{syk}}{\gamma_s E_s} \quad \textbf{(5.13)}$$

Let N_c and N_s refer to the compressive forces in the concrete and steel blocks respectively. Similarly M_c and M_s refer to the moments in the two blocks. Then the integration equations are

$$N_c = 2rt(1-\rho)\int_{\alpha_0}^{\pi} f_{pc}(\varepsilon)d\alpha \quad \textbf{(5.14)}$$

$$M_{uc} = 2r^2t(1-\rho)\int_{\alpha_0}^{\pi} f_{pc}(\varepsilon)\cos(\alpha)d\alpha \quad \textbf{(5.15)}$$

$$N_s = 2rt\rho\int_{0}^{\pi} f_s(\varepsilon)d\alpha \quad \textbf{(5.16)}$$

$$M_{us} = 2r^2t\rho\int_{0}^{\pi} f_s(\varepsilon)\cos(\alpha)d\alpha \quad \textbf{(5.17)}$$

But it is not necessary to calculate the value of the whole normal force or the moment. It is only required to calculate the value of the non dimensional parameters. Using the relations given in equation 5.6 and 5.7 we have

$$n_c = \frac{2(1-\rho)}{f'_{ck}} \int_{\alpha_0}^{\pi} f_{pc}(\varepsilon) d\alpha \tag{5.18}$$

$$m_c = \frac{2(1-\rho)}{f'_{ck}} \int_{\alpha_0}^{\pi} f_{pc}(\varepsilon) \cos(\alpha) d\alpha \tag{5.19}$$

$$n_s = \frac{2\rho}{f'_{ck}} \int_{0}^{\pi} f_s(\varepsilon) d\alpha \tag{5.20}$$

$$m_s = \frac{2\rho}{f'_{ck}} \int_{0}^{\pi} f_s(\varepsilon) \cos(\alpha) d\alpha \tag{5.21}$$

Note that α_0 is the parameter for varying the location of the neutral axis.

These four equations form the basis for the calculation of the interaction curves shown above.

5.6 Conclusions

The stress-strain curves of the steel and the special curve for concrete were formed and justified. The ultimate strength equation was formulated. The interaction curve between moment and compressive force was calculated and plotted. The necessary equations for the same were also derived and listed.