Chapter 4. Estimation of Temperature Load Effects

4.1 Introduction

In walls of Reinforced Concrete chimney, stresses are developed due to the temperature difference between the inner and the outer surface of the walls. This temperature difference from inside to outside tends to expand the inner surface relative to the outer one. Due to the monolithic action of the entire wall, differential expansion is not possible and hence equal expansion takes place so that the shell is compressed on its inside surface and pulled on its outside surface. As a whole there is an average increase in length of the chimney due to the temperature gradient.

Various codes given different methods for the evaluation of the resultant temperature stresses. The CICIND code does not explicitly give equations for the evaluation of these stresses. Instead it asks the designer to account for them assuming the shell wall to be a straight Reinforced Concrete wall.

The ACI code gives a code that is shortly discussed. There is another method that is discussed by the book 'Advanced Reinforced Concrete Design' by Dr. N.Krishna Raju which will be used to calculate the stresses on a typical cross section.

The temperature stresses are of two different types. The stresses that occur in the vertical part of the cross section and the stresses that occur in the horizontal part of the cross section. Also calculations must be performed for the steel on the inside face as well as the outside face of the chimney.

The ACI code gives the following equations to calculate the maximum vertical stresses occurring in steel at the inside of the chimney due to temperature difference. Note that f''_{CTV} refers to the concrete stresses and the value f''_{STV} refers to the stress in steel.

$$f''_{CTV} = \alpha_{te} \bullet c \bullet T_x \bullet E_c \quad \textbf{(4.1)}$$

And

$$f''_{STV} = \alpha_{te} \bullet (c - 1 + \gamma_2) \bullet T_x \bullet nE_c \quad \textbf{(4.2)}$$

Where the terms are explained below

α_{te} = thermal coefficient of expansion of the concrete and of the reinforcing steel, to be taken as 0.0000065 per deg F

E_c = modulus of elasticity of concrete

c is given by the equation

$$c = -\rho n(\gamma_1 + 1) + \sqrt{[\rho n(\gamma_1 + 1)]^2 + 2\rho n[\gamma_2 + \gamma_1(1 - \gamma_2)]} \qquad \textbf{(4.3)}$$

ρ = ratio of the total area of the vertical outside face reinforcement to total area of concrete chimney shell at the section under consideration

γ_1 = ratio of the inside face vertical reinforcement area to the outside face vertical reinforcement area.

γ_2 = ratio of distance between inner surface of chimney shell and center line of outer face vertical reinforcement to total shell thickness

$n = E_s/E_c$ **(4.4)**

T_x is the temperature gradient across the shell.

The code gives a number of formulas for the calculation of this gradient depending on the type of shell. The shell type could be any of unlined chimneys, lined chimneys with insulation completely filling the space between the lining and the shell, lined chimneys with unventilated air space between the lining and the shell or lined chimneys with ventilated space between the lining and the shell.

The equation for the unlined case is given

$$Tx = \frac{td_{ci}}{C_c d_c}\left[\frac{T_i - T_o}{\frac{1}{K_i} + \frac{td_{ci}}{C_c d_c} + \frac{d_{ci}}{K_o d_{co}}}\right] \qquad \textbf{(4.5)}$$

Where the factors are dependant on the cross section under consideration.

The terms K_o and K_i are the coefficients for transfer of heat. These can be obtained from curves given by the code.

The maximum stress in the vertical steel f_{stv} occurring at the outside face of the chimney shell due to the temperature gradient can be computed using

$$f_{STV} = \alpha_{ie} \bullet c' \bullet T_x \bullet E_c \qquad \textbf{(4.6)}$$

An additional kind of temperature stress that is taken into account by the ACI code is the circumferential temperature stress. The equation for the evaluation of the same is

$$f''_{CTC} = \alpha_{ie} \bullet c' \bullet T_x \bullet E_c \tag{4.7}$$

and the same for steel is

$$f''_{STC} = \alpha_{ie} \bullet (\gamma_2 ' c') \bullet T_x \bullet E_s \tag{4.8}$$

4.2 Equations for evaluation of stresses

The following is a derivation of the equations for the temperature stresses. Assume that

T^o is the temperature difference between inside and outside with a linear temperature gradient.

α is the coefficient of expansion of steel and concrete.

e is the strain difference in temperature

m is the modular ratio

t_s is the area of reinforcement per unit width

t_c is the area of concrete per unit width

σ_{ct} is the stress in concrete due to temperature

σ_{st} is the stress in steel due to temperature

p is (t_s/t_c)

k is the neutral axis depth constant.

Referring to the figures below and considering the force equilibrium we have

$$\frac{1}{2}\sigma_{ct} k t_c = t_s \sigma_{st} = p t_c \sigma_{st} \tag{4.9}$$

Which gives on solving for the stress in steel

$$\sigma_{st} = \left(\frac{\sigma_{ct}}{2p}\right) = m\sigma_{ct}\left(\frac{at_c - kt_c}{kt_c}\right) = m\sigma_{ct}\frac{(a-k)}{k} \tag{4.10}$$

The following figures are a representation of the case

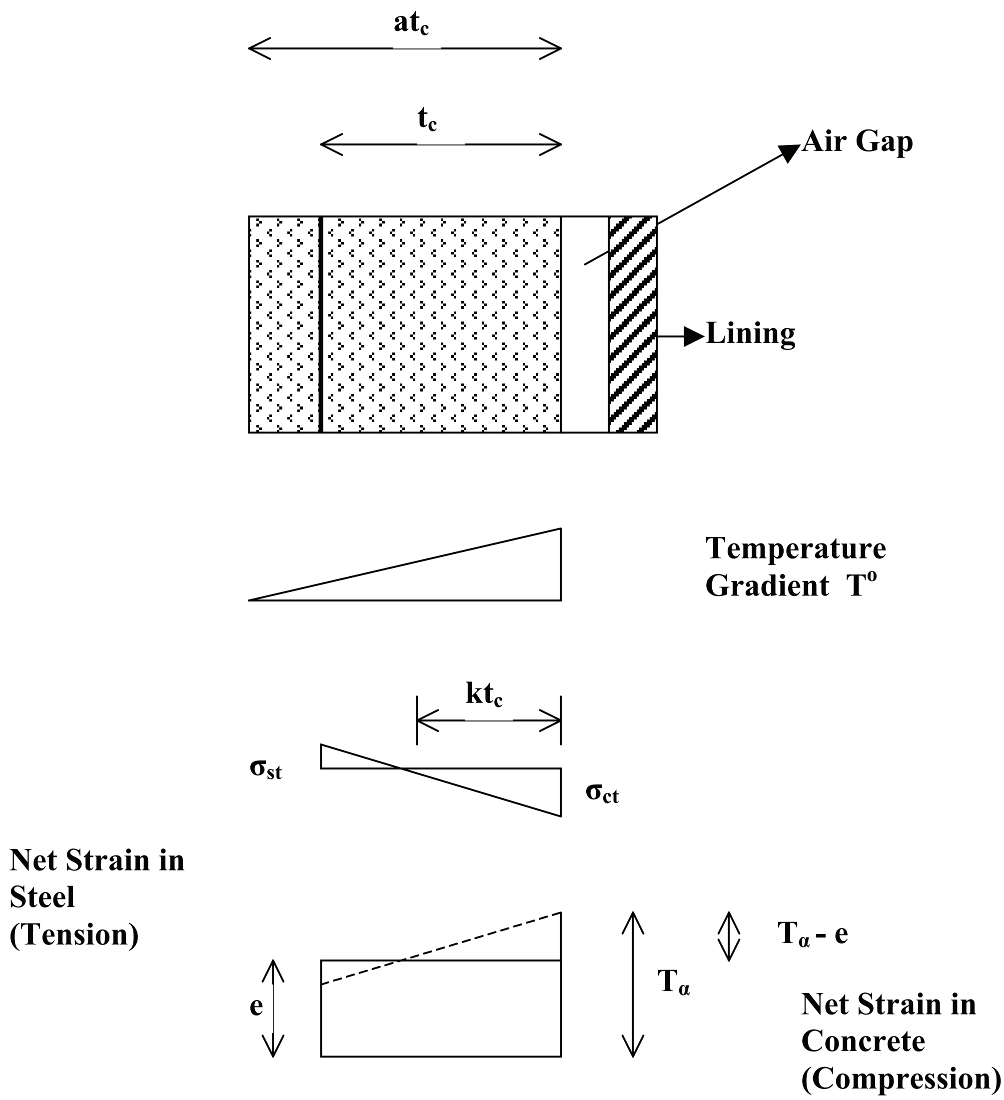

Figure 4.1 – Thermal Stresses

The expressions for stress in steel in turn give the following equation for the value of k^2

$$k^2 = 2pm(a - k) \qquad \textbf{(4.11)}$$

Wherein the value of k is

$$k = -mp + \sqrt{2mpa + p^2m^2} \qquad \textbf{(4.12)}$$

Rise in temperature in reinforcement is

$$(1-a)T \tag{4.13}$$

Free expansion of steel is

$$(1-a)\alpha T \tag{4.14}$$

The tensile stress due to the difference between that due to strain e and due to temperature rise (1-a)T

Hence the stress in steel is

$$\sigma_{st} = Es\left[(1-k)\alpha T - (1-a)\alpha T\right] \tag{4.15}$$

or

$$\sigma_{st} = Es\alpha T(a-k) \tag{4.16}$$

similarly stress in concrete is given by

$$\sigma_{ct} = Ec\alpha kT \tag{4.17}$$

Stresses in horizontal reinforcement

At high temperatures, the inner surface of the chimney is prevented from expansion and therefore gets compressed. The outer surface will expand more than the natural expansion and will be in tension. Due to temperature stresses, generally the hoop tries to expand and consequently tensile stresses develop in the hoop reinforcement.

Using the above figures and the following notation

$k't_c$ = position of the neutral axis

σ'_c = compressive strength in concrete

σ'_s = compressive strength in steel

A_s' = area of hoop reinforcement per unit height

A_s = cross sectional area of steel

The equations for the calculation of the stresses are given as

$$\sigma'_s = m\sigma'_c \frac{(a-k')}{k'} \quad \textbf{(4.18)}$$

$$k' = \sqrt{2pma + p^2m^2} - pm \quad \textbf{(4.20)}$$

$$[\sigma'_s + m\sigma'_c] = E_s\alpha Ta \quad \textbf{(4.21)}$$

Knowing the value of k' the stresses can be calculated.

Sample calculations

The following is a sample calculation for a simple 4000mm concrete Reinforced Concrete wall. The derivation does not take into account the curvature of the shell directly. Hence they can also be applied to any wall. Also the assumed thickness of the wall is quite typical of the chimneys looked into so far.

$t_c = 4000$

assuming a steel cover of about 100 mm

$at_c = 3900$

hence a = 3900/4000 = 0.975

assuming a 1% steel reinforcement

p = 0.01

assume a temperature difference of 75^oC

other values are

$\alpha = 11*10^{-6}$ /^oC

m = 11

$E_s = 210000$

$E_c = 19090.9$

Calculating the value of k' using equation 4.20

k' = 0.366025

Hence the vertical stresses are calculated to be

σ_c	5.765 N/mm^2
σ_s	105.5 N/mm^2

Table 4.1 – Vertical Stresses

The hoop stresses are calculated by solving the following equations

$$\sigma'_s = 18.3\sigma'_c \quad \textbf{(4.22)}$$

And

$$\sigma'_s + 11\sigma'_c = 168.9 \quad \textbf{(4.23)}$$

Wherein the solutions are

σ_c	5.764 N/mm^2
σ_s	105.49 N/mm^2

Table 4.2 – Hoop Stresses

4.3 Conclusions

The cause for the occurrence of thermal stresses in chimney shells were studied. Equations describing the phenomena were derived and stress resultants related. The thermal stresses in the cross section were calculated.